Parameter Extraction and Correction for Transmission Lines and Discontinuities Using the Finite-Difference Time-Domain Method

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Abstract— **The finite-difference time-domain (FDTD) method is useful for performing broadband characterization of uniform transmission lines and discontinuities. Modeling a geometry often requires the implementation of an absorbing boundary condition (ABC). When this is the case, numerical reflections from the ABC's will add significant error to the calculated transmission line or scattering (S) parameters. This paper introduces a simple post-processing algorithm for extracting these parameters and correcting for numerical reflection error. Furthermore, this method is shown to have a unique relationship to Prony's method. Practical application and limitations of this technique are also discussed. Finally, the impedance and propagation constant of a miscrostrip line are calculated using this method.**

I. Introduction

APPLICATION of the finite-difference time-domain (FDTD) method [1] to calculate transmission line and scattering (S) parameters provides broadband frequency information in a single simulation. This paper first focuses on analysis of a uniform cross-section line, and then extends the approach to computing S-parameters for an arbitrary discontinuity. Modeling a uniform line in the time-domain involves creating a two-dimensional (2-D) rectangular mesh for the transverse cross section and extruding the mesh in the longitudinal direction. Next, a TEM or quasi-TEM mode is excited on the structure. The voltage and current along the line are then calculated from the time-domain fields. Practical application of this standard technique necessitates the use of an absorbing boundary condition (ABC). Even if the cross section is closed (PEC and PMC boundaries only), it is still necessary to terminate the line with an ABC. Most ABC's are derived by making an approximation to an outward-propagating wave equation using the electromagnetic field data in neighboring cells around each boundary node. Although this approach works well, residual reflections do arise from these ABC's. This paper utilizes the standard first order Mur ABC [2] due to its simplicity, reasonable performance, and general acceptance.

The ABC reflections give rise to standing waves on a transmission line, rather than a traveling wave in a single direction. In order to correct for such errors, it is necessary to separate the standing wave into forward and backward traveling wave components. Other techniques have been suggested in the past [3]–[4], but are more complicated than the algorithm presented below. The method used in [3] involves computation of Fourier series and iterative solution of a nonlinear matrix equation to calculate the ABC reflection. The approach used in [4] is similar to the one presented below, but involves matrix inversion. The method developed here only assumes that the voltage and current on a transmission line are a superposition of TEM waves. The reflection is calculated in closed form, involving no matrix equations. As will be evident, this method is very general and has direct application to computing S-parameters.

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II. Theory

The derivation for the error correction is a very straightforward manipulation of the transmission line equations. Assume that the voltage and current at any point on a transmission line are calculated from an FDTD simulation, and then Fourier transformed into the frequency domain. At any frequency, these quantities can be represented by the following equation:

$$f(x) = f_x = c_1 u^x + c_2 u^{-x}. \tag{1}$$

The variable f is either voltage or current, x is distance along the transmission line, c_1 and c_2 are the coefficients for the positive and negative traveling waves, and u is the base of the complex wave function, $e^{-\gamma}$. Sampling at three uniform points with spacing d on the transmission line produces three equations in three unknowns, which are u, c_1, and c_2

$$f_0 = c_1 + c_2 \tag{2a}$$

$$f_1 = c_1 u^d + c_2 u^{-d} \tag{2b}$$

$$f_2 = c_1 u^{2d} + c_2 u^{-2d}. \tag{2c}$$

Eliminating c_1 and c_2 in (2c) and simplifying yields the following quadratic in u^d

$$u^{2d} - \frac{f_0 + f_2}{f_1} u^d + 1 = 0. \tag{3}$$

The constant term of one in this equation implies that the roots are reciprocals of one another. This agrees with physical intuition since we expect two waves that have mutually reciprocal wave functions. The two solutions are given in (4)

$$u^d = \left(\frac{f_0 + f_2}{2f_1}\right) \pm \sqrt{\left(\frac{f_0 + f_2}{2f_1}\right)^2 - 1}. \tag{4}$$

The wave coefficients, c_1 and c_2, can be expressed in terms of u as follows:

$$c_1 = \frac{f_0 - f_1 u^d}{1 - u^{2d}} \tag{5a}$$

$$c_2 = \frac{f_0 - f_1 u^{-d}}{1 - u^{-2d}}. \tag{5b}$$

Note that these two equations yield the same set of coefficients regardless of which root is selected for performing the calculations. This is evident from the reciprocal nature of the two roots and the high degree of symmetry in (5a)–(5b). Note that the data plane for f_0 is the phase reference for the coefficients. Determining a sense of direction (forward or backward) depends on the propagation constant and the assumed time convention.

This derivation can now be applied to calculating the transmission line parameters of an arbitrary uniform line using the following procedure. First, apply the above procedure independently to the voltage and current signals at three regular points on the line. Examination of the sign of the imaginary part of the exponent determines the direction of wave propagation. The complex line impedance is given by the ratio of the voltage to current for the forward traveling wave, or the negative of this ratio for the backward traveling wave

$$Z = \frac{V_+}{I_+} = -\frac{V_-}{I_-} = \frac{c_v^+}{c_i^+} = -\frac{c_v^-}{c_i^-}. \tag{6}$$

A more accurate criteria for computing impedance is to take the ratio using the set of voltage and current coefficients with the larger overall magnitude, and toggle the sign of the impedance such that the real part is positive. The propagation constant is somewhat more difficult to extract since it involves taking the natural log of a complex number. It is easier to separately calculate the real and imaginary parts of the complex propagation constant. The attenuation constant is readily found from the u^d term as follows:

$$\alpha = \frac{|\mathrm{Re}\{\ln u^d\}|}{d}. \tag{7a}$$

The absolute value sign makes this calculation independent of root selection. The phase constant is calculated in a similar manner as shown below:

$$\beta = \frac{|\mathrm{Im}\{\ln u^d\}| + 2\pi n}{d}. \tag{7b}$$

The $2\pi n$ term is necessary to account for the branch cut of the complex log function. The phase constant is linear with frequency and zero at dc. The correct value of n can be tracked by using an initial value of zero and incrementing it by one every time the branch cut is crossed. Since the branch cut for the complex log function occurs at the negative real axis, crossings can be indicated by a sign change in the imaginary part of $\mathrm{In}(u^d)$. The direction of the sign change depends on the root selection. A wave function of the form $e^{-\gamma d}$ crosses the branch cut from negative to positive imaginary part, while $e^{+\gamma d}$ crosses from positive to negative. Fortunately, this issue can usually be ignored by proper selection of the spacing d. As long as βd is bound by 0 and π, a branch cut is not encountered. This implies the following constraint on d

$$d_{\max} \leq \frac{\pi}{\beta_{\max}} = \frac{\pi}{2\pi/\gamma_{\min}} = \frac{\gamma_{\min}}{2}. \tag{8}$$

The periodicity of voltage and current along a transmission line every half wavelength dictates using a maximum d of a quarter wavelength in order to guarantee a nonsingular system of equations in (2a)–(2c). Therefore, a proper choice of the spacing d eliminates the branch cut problem. The optimum value of d is obviously an eighth of a wavelength. Finally, the complex propagation constant can then be expressed as follows:

$$\gamma = \frac{|\mathrm{Re}\{\ln u^d\}|}{d} + j\frac{|\mathrm{Im}\{\ln u^d\}|}{d}. \tag{9}$$

The RLGC parameters are readily computed from γ and Z using the relations

$$R + j\omega L = \gamma Z \tag{10a}$$

$$G + j\omega C = \gamma / Z. \tag{10b}$$

This method is easily extended to the calculation of scattering parameters, since that basically involves breaking a signal into positive and negative directed wave components. This avoids running an additional simulation to compute the incident voltage or current signal.

For an arbitrary N-port network, one column of an S-Matrix can be computed using a single FDTD simulation. The excitation is applied at a single port in the device. Using the outlined technique, the incident and reflected signals are computed at the excited port, as well as the transmitted signals at the other ports. This is the common approach to calculating S-parameters. The extraction technique eliminates the need for an extra FDTD simulation to compute the incident signal. This efficient procedure is very similar to the one presented in [4], but note that neither method corrects for reflections from boundaries when calculating S-parameters. A general method for performing such corrections is not easily derived. However, a good approximation utilizing the above technique can be developed for symmetrical, reciprocal two-port devices. First consider the following one-port example. The voltage on a transmission line can be expressed as

$$V(z) = \frac{Z_0 V_S}{Z_0 + Z_S} \frac{e^{-\gamma d}}{1 - \Gamma_S \Gamma_L e^{-2\gamma d}} (e^{+\gamma z} + \Gamma_L e^{-\gamma z}) \tag{11}$$

where the S and L subscripts refer to the source and load respectively. Equation (11) assumes that the load is at $z = 0$ and the source is at $z = -d$. Note that the ratio of the reflected to incident waves is the load reflection coefficient, and it is independent of source mismatch. This implies that the extraction procedure outlined above will work exactly for characterizing uniform transmission lines and one-port discontinuities. In this case, one imperfect ABC represents source mismatch, and the other imperfect ABC defines the load reflection coefficient. This same line of reasoning is applicable to an arbitrary two-port network as shown in Fig. 1.

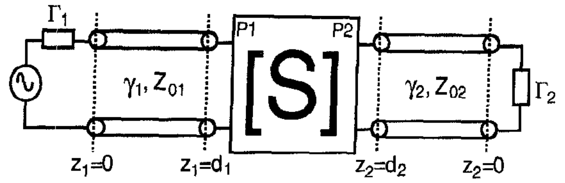

Fig. 1. Generalized two-port network.

First-order approximations for the line voltage at each port can be expressed as follows:

$$\begin{aligned} V_1(z_1) &= (1+\Gamma_1 S_{11} e^{-2\gamma_1 d_1}) e^{-\gamma_1 z_1} \\ &\quad + (S_{11} e^{-2\gamma_1 d_1}(1+\Gamma_1 S_{11} e^{-2\gamma_1 d_1}) \\ &\quad + \Gamma_2 S_{12} S_{21} e^{-2\gamma_1 d_1} e^{-2\gamma_2 d_2}) e^{+\gamma_1 z_1} \end{aligned} \tag{12a}$$

$$\begin{aligned} V_2(z_2) &= \sqrt{\frac{Z_{02}}{Z_{01}}} S_{21} e^{-\gamma_1 d_1} (1+\Gamma_1 S_{11} e^{-2\gamma_1 d_1} \\ &\quad + \Gamma_2 S_{22} e^{-2\gamma_2 d_2}) e^{-\gamma_2 z_2} \\ &\quad + \sqrt{\frac{Z_{02}}{Z_{01}}} (S_{21} e^{-\gamma_1 d_1} \Gamma_2 e^{-2\gamma_2 d_2}) e^{+\gamma_2 z_2}. \end{aligned} \tag{12b}$$

First-order implies the inclusion of all terms involving ABC reflection coefficients of order one. Using the outlined extraction method, the constants in the following generic representation can be computed:

$$V_1(z_1) = c_1^+ e^{-\gamma_1 z_1} + c_1^- e^{+\gamma_1 z_1} \tag{13a}$$

$$V_2(z_2) = c_2^+ e^{-\gamma_2 z_2} + c_2^- e^{+\gamma_2 z_2}. \tag{13b}$$

Equations (12a)–(12b) and (13a)–(13b) imply the following conditions on these constants:

$$c_1^+ e^{-2\gamma_1 d_1} S_{11} + c_2^- e^{-\gamma_1 d_1} \sqrt{\frac{Z_{01}}{Z_{02}}} S_{12} = c_1^- \tag{14a}$$

$$c_1^+ e^{-\gamma_1 d_1} \sqrt{\frac{Z_{01}}{Z_{02}}} S_{21} + c_2^- S_{22} = c_2^+. \tag{14b}$$

Assuming symmetry and reciprocity, the full S-matrix can be derived from (14a–14b) as shown:

$$S_{11} = S_{22} = \frac{c_1^+ c_1^- Z_{02} - c_2^+ c_2^- Z_{01}}{(c_1^+)^2 e^{-2\gamma_1 d_1} Z_{02} - (c_2^-)^2 Z_{01}} \tag{15a}$$

$$S_{12} = S_{21} = \sqrt{Z_{01} Z_{02}} \frac{c_1^+ c_2^+ e^{-\gamma_1 d_1} - c_1^- c_2^- e^{+\gamma_1 d_1}}{(c_1^+)^2 e^{-2\gamma_1 d_1} Z_{02} - (c_2^-)^2 Z_{01}}. \tag{15b}$$

Although S-parameter calculations are not as susceptible to ABC reflections as transmission line parameters, this approach proves to be superior to the standard technique especially for broadband frequency analysis. Note that a similar procedure using currents could also be derived. Additionally, more sophisticated procedures could be applied to better characterize the boundary reflections, thus improving this calibration procedure.

III. Application

The above procedure has proven to be simple to implement in software, has provided fast accurate results, and has general applicability to similar and more advanced types of problems. The best way to gain experience with this technique is to experiment with different ways of performing these calculations. For example, the impedance can be calculated with the dominant traveling wave, or from the reflected wave. In this case, it is not recommended that the reflected wave be used in the calculation since ratios of small magnitude numbers are numerically less accurate. Based on detailed investigation, the following suggestions are offered.

For best results in the time domain, spatially excite the line with the dominant TEM or quasi-TEM mode as well as possible. Use a smooth temporal excitation, such as a Gaussian pulse or Blackman-Harris window, allowing enough iterations for the fields to settle to their initial unexcited state. Remove the inherent half-cell spatial separation between voltage and current by averaging. Account for the half-time step difference between E and H by using the appropriate scaling factor of $e^{\pm j\omega\Delta t/2}$. Before performing a Fourier transform into the frequency domain, remove any residual dc offset in the voltage and current time signatures. Since the offset is usually not constant, subtract a linear approximation to this offset. In other words, subtract from it a line passing through the first and last data points. This amounts to rotating the time signature toward the axis by the arctangent of the slope of the linear offset. Although this offset is usually quite small, it is a good practice to remove it [5]. The voltage and current signatures are ready to be transformed into the frequency domain via a fast Fourier transform (FFT). Conservative handling of the time domain data helps ensure a smooth transition into the frequency domain, and significantly improves results toward dc.

Next, the wave decomposition algorithm can be applied. This requires at least three voltage and current samples. If $n \geq 3$ samples are available, then the linear coefficient in the quadratic equation (3) can be averaged over the $(n-2)$ data triplets. No other modification is necessary, and one only needs to substitute the averaged coefficient when evaluating (4). For the purpose of calculating these roots, the current data usually provides the best results. Using this set of roots, the coefficients for both the voltage and current waves can be calculated by using (5a)–(5b). The roots computed from the voltage are usually noisier than those for current, and once again, some experimentation is necessary to determine the better choice. The impedance can then be calculated as the ratio of the dominant voltage and current wave coefficients with an appropriate sign adjustment. The propagation constant is readily calculated from (9), which is independent of the root selection.

It should be pointed out that the method outlined above has two practical limitations. At high frequencies, the spacing d will become greater than a quarter wavelength making the system of equations (2a)–(2c) potentially ill-conditioned. At low frequencies, the spacing d becomes small compared to a wavelength, making the system of equations increasingly ill-conditioned. Ideally, the sampling points should be equally spaced within a quarter wavelength at the center frequency of analysis. An analytical approach can be employed to determine the bandwidth over which the above calculation is well-behaved. If bandwidth limitations arise, the simple solution is to split the frequency band into smaller subbands and select an

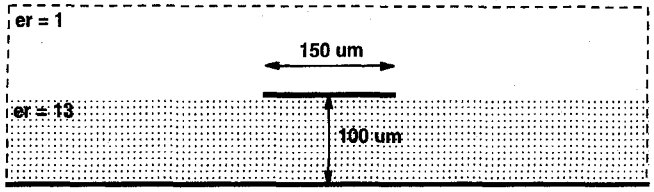

Fig. 2. Microstrip geometry.

appropriate spacing d for each subband. Obviously, the spacing decreases for each higher frequency band. This algorithm is clearly capable of handling most any transmission line or discontinuity problem at any frequency. Furthermore, this method works well quite independent of the ABC performance, which is the most powerful characteristic of the given formulation.

IV. Relationship to Prony's Method

The equations derived above have a special relationship to Prony's method, which is a more general exponential approximation to a function. The concept behind Prony's method is to approximate a function as a sum of complex exponentials as shown below:

$$f(x) = f_x = \sum_{i=1}^{n} a_i u_i^x = a_1 u_1^x + \cdots + a_n u_n^x. \tag{16}$$

Assume that the function f is known at $2n$ equally spaced points which can be linearly mapped to the set of integers $x \in \{0, 1, \cdots, 2n-1\}$. This data could be used to generate a nonlinear system of $2n$ equations; however, it would be very difficult to solve. Prony's method simplifies this problem by solving it in three stages: two linear and one nonlinear. Substituting the first n data samples into (16) yields the following linear system of n equations:

$$\begin{pmatrix} 1 & \cdots & \cdots & 1 \\ u_1 & \cdots & \cdots & u_n \\ \cdots & \cdots & \cdots & \cdots \\ u_1^{n-1} & \cdots & \cdots & u_n^{n-1} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \cdots \\ a_n \end{pmatrix} = \begin{pmatrix} f_0 \\ f_1 \\ \cdots \\ f_{n-1} \end{pmatrix}. \tag{17}$$

If the elements in vector u are available, then vector a is readily found using a matrix inversion scheme. The vector u can be solved for as follows. Let the elements in u be the n roots of the polynomial equation given below:

$$u^n + r_1 u^{n-1} + \cdots + r_{n-1} u + r_n = 0. \tag{18}$$

Another linear system of n equations can be generated using the $2n$ data samples and (18). The general form of this system is as follows:

$$\begin{pmatrix} f_{n-1} & f_{n-2} & \cdots & f_0 \\ f_n & f_{n-1} & \cdots & f_1 \\ \cdots & \cdots & \cdots & \cdots \\ f_{2n-2} & f_{2n-3} & \cdots & f_{n-1} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ \cdots \\ r_n \end{pmatrix} = - \begin{pmatrix} f_n \\ f_{n+1} \\ \cdots \\ f_{2n-1} \end{pmatrix}. \tag{19}$$

Each equation in (19) is a linear combination of f_x evaluated at different points. The first equation represents a summation of r_n times f_0, r_{n-1} times $f_1, \cdots, r_1$ times f_{n-1}, and 1 times f_n. Expanding each value of f_x using (16), rearranging terms,

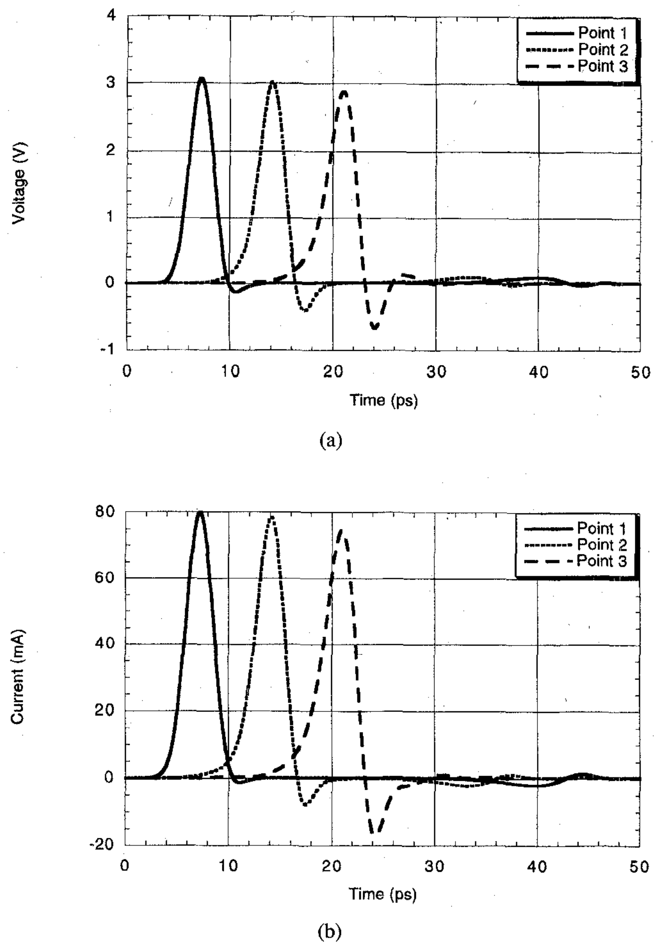

Fig. 3. Time signatures. (a) Voltage. (b) Current.

and applying (18) indicates that this sum adds to zero, thus validating the first equation in (19). The other equations behave likewise. Specifically, these sums add to zero because each can be rearranged as follows:

$$\begin{aligned} f_{n+p} &+ \sum_{i=1}^{n} f_{n-i+p} r_i \\ &= \sum_{i=1}^{n} c_i u_i^p (u_i^n + r_1 u_i^{n-1} + \cdots + r_{n-1} u + r_n) = 0. \end{aligned} \tag{20}$$

The parameter p varies from 0 to $n-1$, specifying one of the equations in (19). Since each u_i is a root, (20) states that the last term in the right summation is zero. Vector r can now be calculated using the data samples and a matrix inversion scheme. Using a polynomial solver, the roots of (18) can be computed. As mentioned before, the coefficients can then be calculated from (17). Prony's method thus explains how to derive an n-term exponential approximation to an arbitrary function. Obviously, a least squares approach can be applied to the matrix equations (17) and (19) if more than n data samples are available.

The method presented in this paper is a special case of Prony's method for a 2-term ($n = 2$) exponential approximation in which the two roots are forced to be reciprocals. For

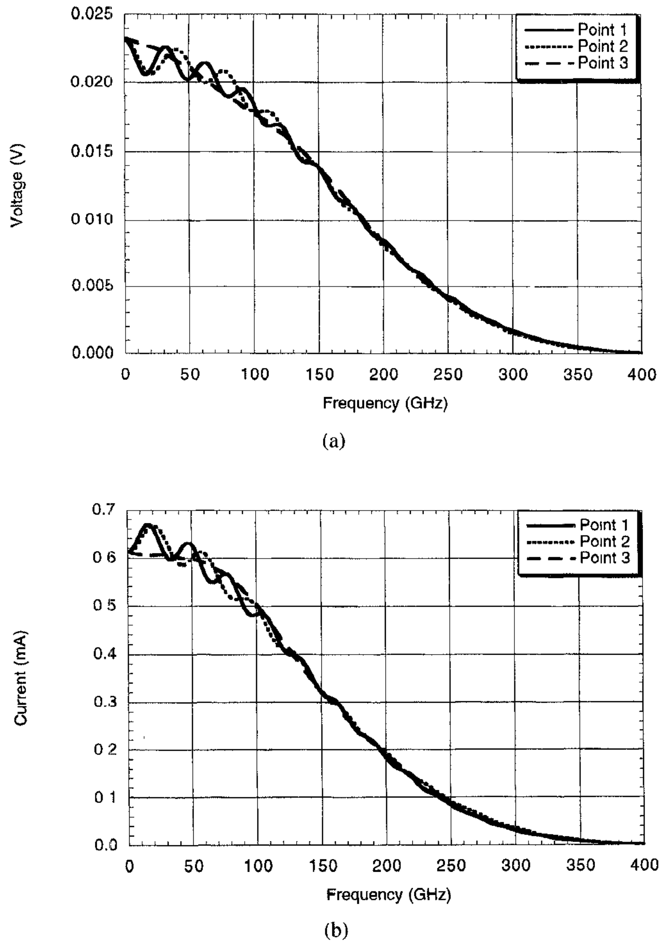

Fig. 4. Magnitude spectra. (a) Voltage. (b) Current.

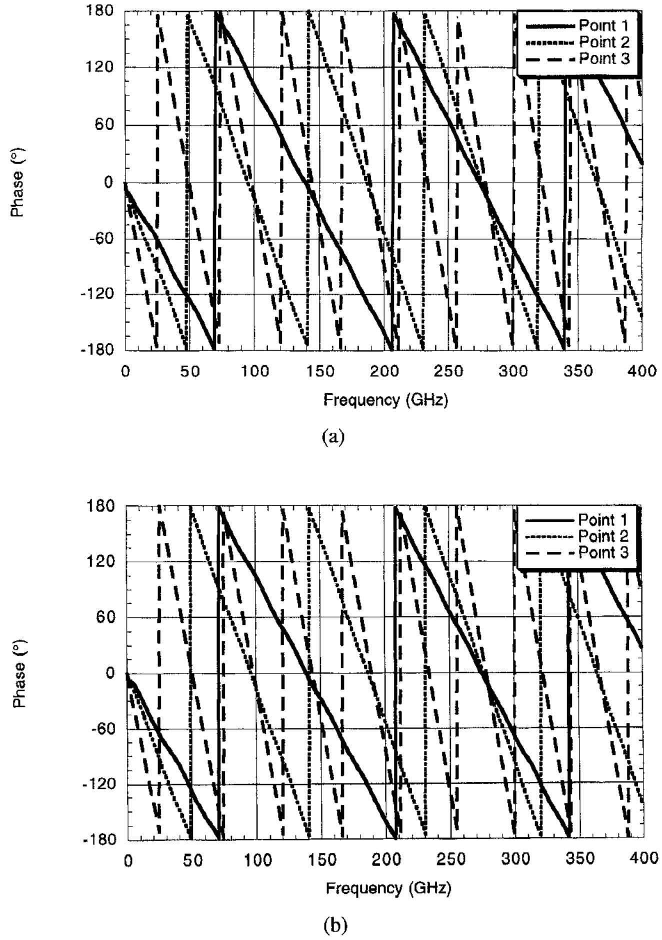

Fig. 5. Phase spectra. (a) Voltage. (b) Current.

an exact solution (no least squares approximation), a total of $2 * n = 4$ samples are required, as shown in (21a)–(21d):

$$f_0 = c_1 + c_2 \tag{21a}$$

$$f_1 = c_1 u_1 + c_2 u_2 \tag{21b}$$

$$f_2 = c_1 u_1^2 + c_2 u_2^2 \tag{21c}$$

$$f_3 = c_1 u_1^3 + c_2 u_2^3. \tag{21d}$$

The matrix equation for the c vector is given as follows:

$$\begin{pmatrix} 1 & 1 \\ u_1 & u_2 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} f_0 \\ f_1 \end{pmatrix}. \tag{22}$$

The characteristic equation for u is the generic quadratic (23):

$$u^2 + r_1 u + r_2 = 0. \tag{23}$$

The r vector is given by the following matrix equation:

$$\begin{pmatrix} f_1 & f_0 \\ f_2 & f_1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = -\begin{pmatrix} f_2 \\ f_3 \end{pmatrix}. \tag{24}$$

An algebraic solution for (22) through (24) is readily obtained, and the resulting solutions for r, u, and c are as follows:

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = -\frac{1}{f_1^2 - f_0 f_2} \begin{pmatrix} f_1 & -f_0 \\ -f_2 & f_1 \end{pmatrix} \begin{pmatrix} f_2 \\ f_3 \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{f_0 f_3 - f_1 f_2}{f_1^2 - f_0 f_2} \\ \dfrac{f_2^2 - f_1 f_3}{f_1^2 - f_0 f_2} \end{pmatrix} \tag{25}$$

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \dfrac{-r_1 + \sqrt{r_1^2 + 4r_2}}{2} \\ \dfrac{-r_1 - \sqrt{r_1^2 - 4r_2}}{2} \end{pmatrix} \tag{26}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \frac{1}{u_2 - u_1} \begin{pmatrix} u_2 & -1 \\ -u_1 & 1 \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{f_1 - f_0 u_2}{u_1 - u_2} \\ -\dfrac{f_1 - f_0 u_1}{u_1 - u_2} \end{pmatrix}. \tag{27}$$

Forcing the roots u_1 and u_2 to be reciprocal is equivalent to setting r_2 to 1, since in a quadratic equation the constant term equals the product of its roots. Setting r_2 to 1, allows for f_3 to be solved for in terms of f_0, f_1, and f_2. This reduces the necessary number of samples from four to three. Substituting f_3 into r_1, allows r_1 to be defined in terms of f_0, f_1, and f_2 only. The resulting expression for r_1 is the familiar $-(f_0 + f_2)/f_1$ coefficient from (3). This establishes the relationship between Prony's method and the method introduced in this paper.

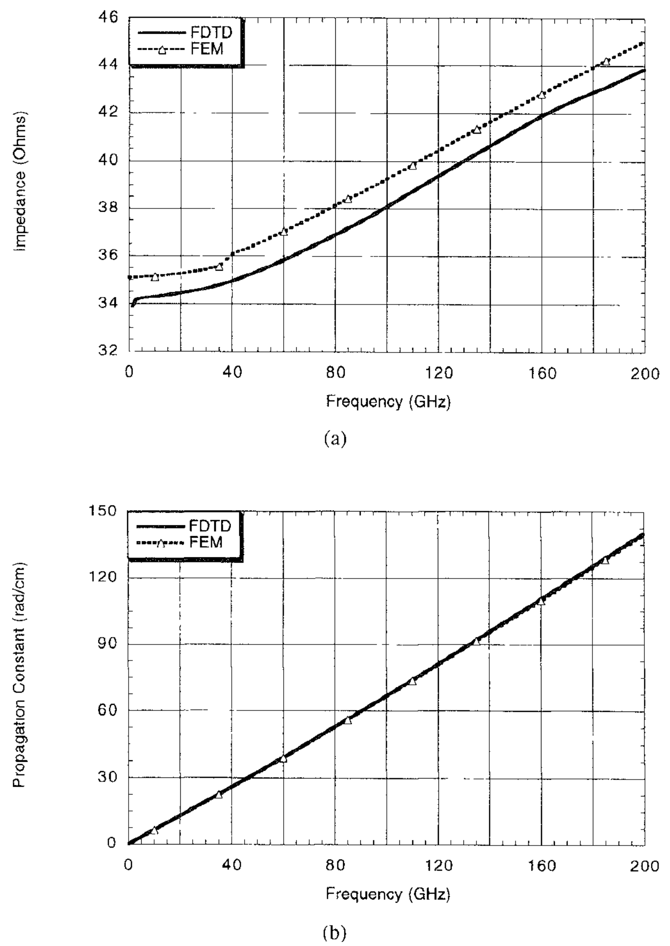

Fig. 6. FDTD and FEM transmission line parameters (a) Impedance. (b) Propagation constant

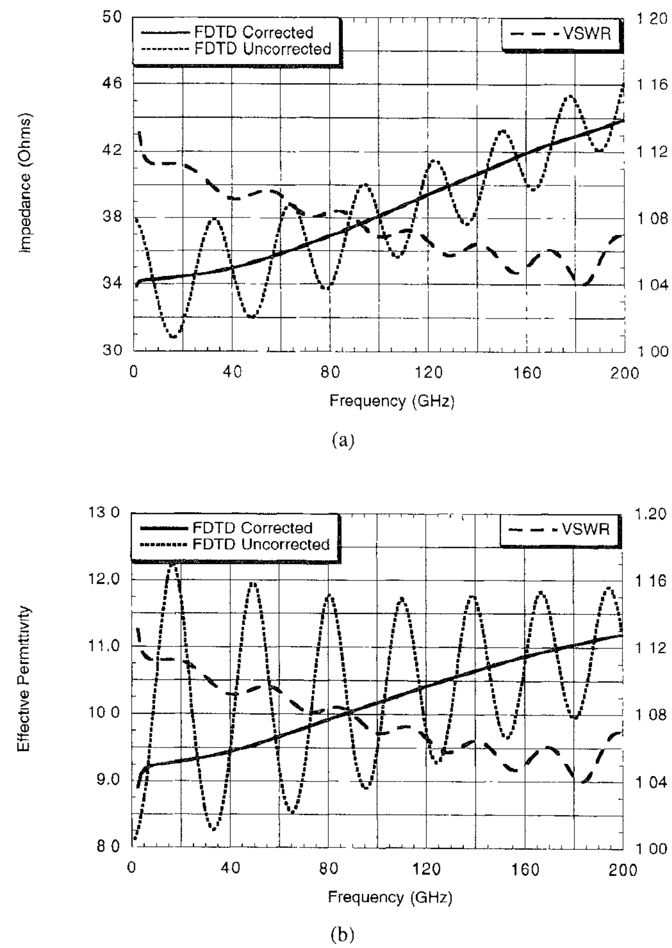

Fig. 7. Corrected and uncorrected transmission line parameters. (a) Impedance and ABC VSWR. (b) Effective permittivity and ABC VSWR.

In practice, the outlined method yields better results than Prony's method for parameter extraction. Prony's method has an extra degree of freedom in the sense that the two waves can have different propagation constants. Physics tells us that the two waves will be identical, except that they propagate in opposite directions. Prony's method is emphasized here since it has useful applications for lines supporting multiple modes of propagation, and thus multiple wave functions. Prony's method could be similarly modified for the multiple mode case as was done for the single mode case.

V. Parameter Extraction Example

The above method is verified with the following microstrip transmission line example. The physical structure is shown below in Fig. 2.

The mesh was generated using a 12.5 μm $\times$ 12.5 μm $\times$ 12.5 μm unit cell. A time step of 23.8 fs was chosen based on the Courant condition. Spatially, the source excites a static field distribution to approximate the quasi-TEM dominant mode. Temporally, the source is defined as a Blackman-Harris window with approximately 200 GHz of bandwidth. Several voltages and currents were monitored along the line, with a uniform spacing of 5 cells or 62.5 μm between points. Fig. 3 shows selected time signatures from this data.

Apart from minor degradation due mostly to physical geometric dispersion, these signals appear to be identical with a uniform delay progressing along the line. The magnitude and phase spectra of these signals are given in Figs. 4 and 5.

As expected, the magnitudes are approximately equal, and the phases have progressively steeper slopes. The transmission line parameters are readily extracted from this data using the outlined technique. Fig. 6 compares these parameters to those obtained using Hewlett-Packard's FEM solver, HFSS [6].

These plots indicate strong agreement between the FDTD and FEM approaches. It is worth mentioning that the computational run times for each solver are on the same order of magnitude. A 2-D FEM solver is fast and efficient; however, to produce the same frequency resolution as FDTD, several frequencies need to be calculated. The final point to make with this example is the necessity of correcting for errors caused by reflections. These reflections produce standing waves on the transmission line. One obvious consequence is that the impedance is no longer independent of position on the line. Fig. 7 below compares these parameters to the parameters calculated with no standing-wave correction.

As can be seen in Fig. 7(a), the uncorrected impedance oscillates around the corrected value as frequency changes. Note that as the VSWR improves, the uncorrected impedance more closely matches the corrected value. Similar effects can be seen in the effective permittivity in Fig. 7(b).

VI. Conclusion

A simple algorithm for extracting transmission line parameters has been introduced. This method has been shown to be quite general and powerful, and considerably easier to implement than other techniques. The method can be derived from physical or mathematical aspects, using either the transmission line equations or Prony's method, respectively. This method was also generalized to S-parameter extraction, specifically for reciprocal and symmetrical two-port networks. Generalization to similar problems was also suggested. Finally, the algorithm was verified on a microstrip line selected from the literature [3]. The transmission line parameters for this geometry were accurately extracted and compared to results from a finite element simulation.

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